Overview of verified quantum computations using MBQC and recent progress

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Outline

1. What is verification and why it is important
2. Short introduction to MBQC
3. Intuition and protocol construction for verified MBQC
4. Abstracting and generalizing
5. Lifting limitations and going toward practical solutions

01

What is verification and why it is important

An (overly simplified) example

Selling the Hadamard gate

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First attempt

- $H\,|0\rangle = |+\rangle \quad H\,|1\rangle = |-\rangle$

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- $H\left|0\right\rangle = \left|+\right\rangle \quad H\left|1\right\rangle = \left|-\right\rangle$
- $\Pr(\left|0\right\rangle \mid H\left|0\right\rangle) = \frac{1}{2}$
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- $H^2\left|\psi\right\rangle = \left|\psi\right\rangle$
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Telling apart genuine and malicious implementations: offline setup

Gate tomography

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Telling apart genuine and malicious implementations: online setup

Limitations to gate tomography:

- There is no guarantee that the behavior of gates will be repeatable
- There is no guarantee that the behavior alone / inside a computation is the same (ie scalability pb)

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- Computation
  - > if $b = 1$ it sends the allowed leakage to the Server, and if $c = 1$ is sent in return it sends $|\perp\rangle\langle\perp| \otimes |\mathrm{Rej}\rangle\langle\mathrm{Rej}|$ to the Client
  - > Otherwise it sends $C(|0\rangle\langle 0|^{\otimes n}) \otimes |\mathrm{Acc}\rangle\langle\mathrm{Acc}|$ to the Client

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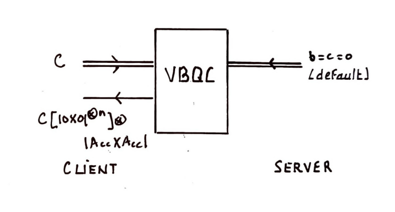
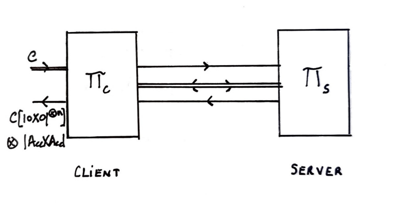
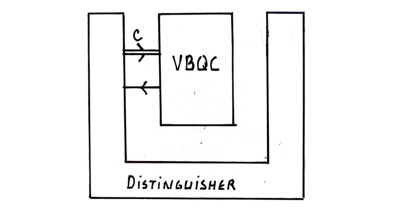
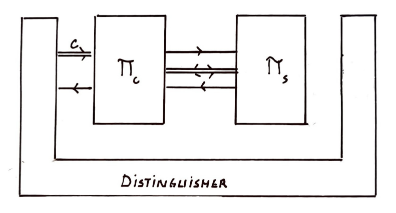


Protocol: Verified and Blind Quantum Computation



Correctness





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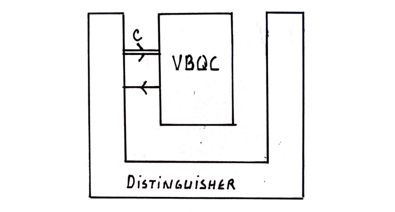
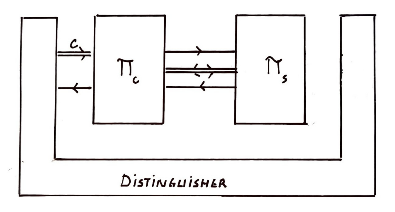
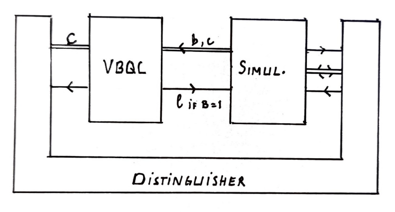
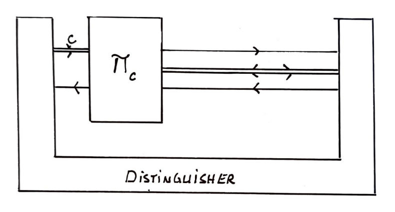




Security

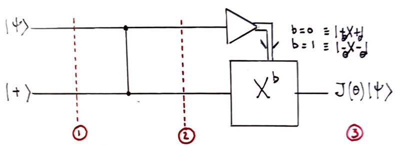




02

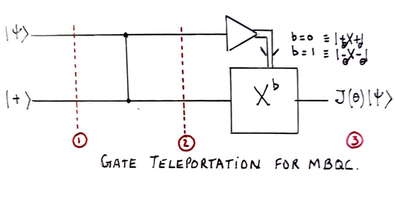
Short introduction to MBQC

Gate Teleportation



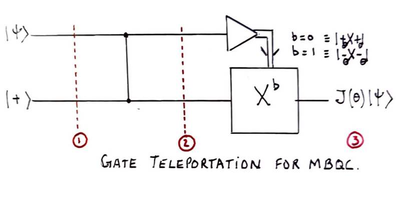
GATE TELEPORTATION FOR MBQC.

Gate Teleportation



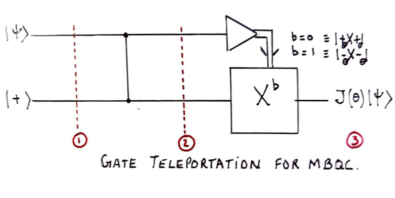
1. $(\alpha |0\rangle + \beta |1\rangle) \otimes |+_\theta\rangle$

Gate Teleportation



1. $(\alpha|0\rangle + \beta|1\rangle) \otimes |+_\theta\rangle$
2. $|+_\theta\rangle \otimes (\alpha|+\rangle + e^{i\theta}\beta|-\rangle)/\sqrt{2} + |-_\theta\rangle \otimes (-\alpha|+\rangle + e^{i\theta}\beta|-\rangle)/\sqrt{2}$

Gate Teleportation



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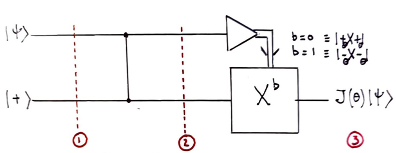
3. $\alpha|+\rangle + \beta e^{i\theta}|-\rangle = HZ(\theta)(|0\rangle + \beta|1\rangle)$

Gate teleportation and universal computation

Gate Teleportation



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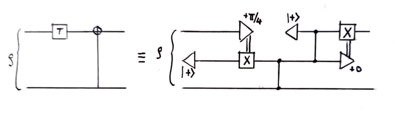
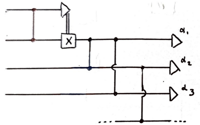
Universality



UNIVERSALITY OF $J(\alpha)$ + C-Z

Pushing corrections to the end



TYPICAL RESULT OF GT COMPILATION

MBQC as lazy implementation of GT

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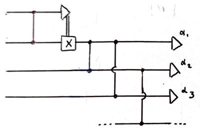

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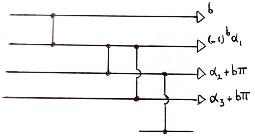
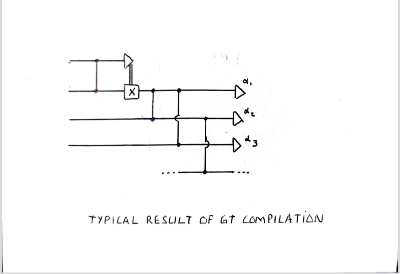
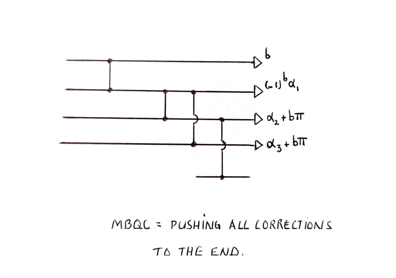


MBQC = PUSHING ALL CORRECTIONS TO THE END.

MBQC as lazy implementation of GT

Pushing corrections to the end





Summary

- Graph and partial order over vertices
- Flow
- Measurement angles (for the all-0 branch)

03

Intuition and protocol construction for verified MBQC

Intuition building

Preventing the Server to be malicious

Efficiency is not guaranteed

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Making sure the Server is caught

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- Constantly test the behavior of the Server

Intuition building

Preventing the Server to be malicious

Efficiency is not guaranteed

Making sure the Server is caught

- Constantly test the behavior of the Server
- Make sure tests and computation look the same

Blindness

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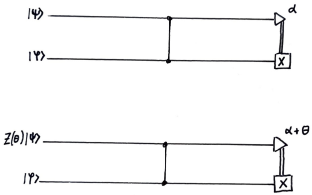
- By sending $|+\rangle_\theta$ the client defines a relative RF unknown to the Server



CONFUSING THE SERVER WITH HIDDEN REL. REFERENCE FRAME

Constructing a verification protocol with MBQC

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- MBQC works because Client and Server share a reference frame
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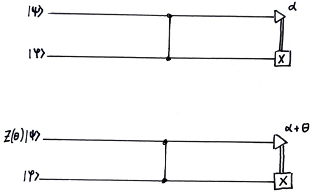
- Blindness reduces attacks to convex combinations of Pauli deviations



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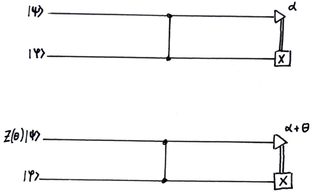

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Trap insertion

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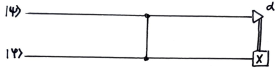
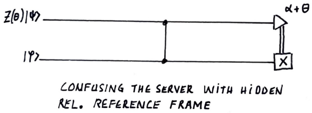
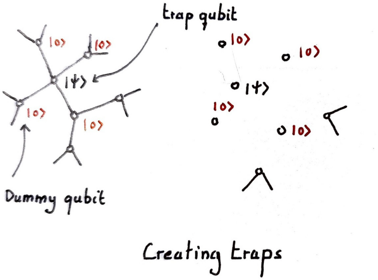
Trap insertion

- Creating traps







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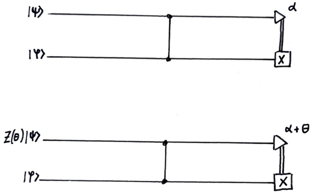

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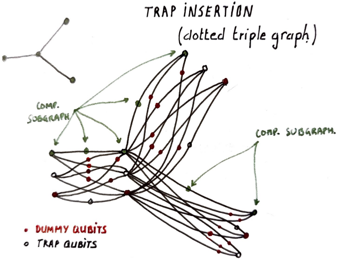

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Trap insertion

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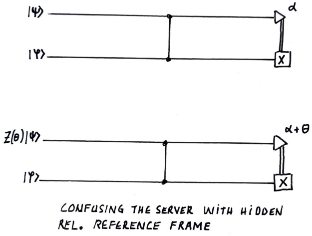
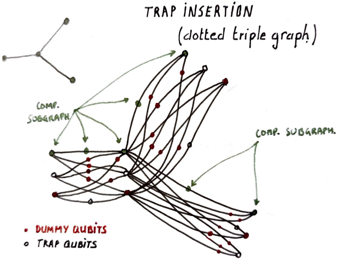
- Allows deviation detection with constant probability





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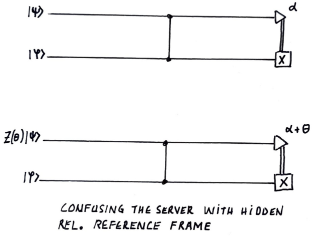

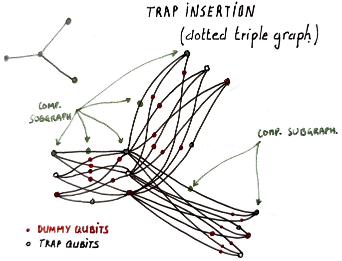

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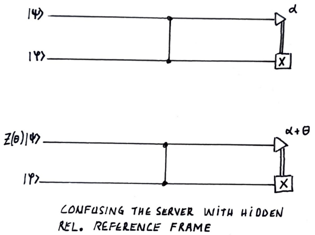
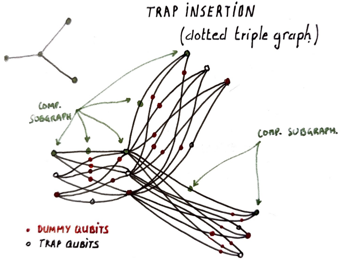
- Making sure that harmfull deviations are detected





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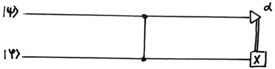
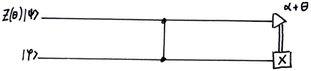
Amplification

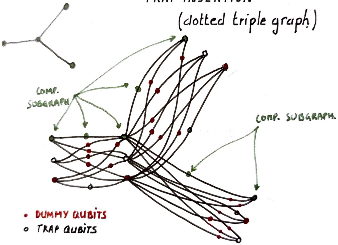
- Making sure that harmfull deviations are detected
- Using fault-tolerant encoding before trap insertion





CONFUSING THE SERVER WITH HIDDEN REL. REFERENCE FRAME

TRAP INSERTION (dotted triple graph)



• DUMMY QUBITS
∘ TRAP QUBITS

Limitations

Overhead

- Fault-tolerant encoding for amplification is costly
- Security competes with computing power (ie. for the number of live-qubits)

Limitations

Overhead

- Fault-tolerant encoding for amplification is costly
- Security competes with computing power (ie. for the number of live-qubits)

Robustness

- The fault-tolerant encoding does not protect from errors
- As soon as a single trap fails, the computation is aborted

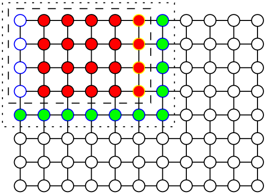
04

Abstracting and generalizing

Trappified canvas

Partial pattern

- G_P subgraph of G
- Input and output sets of nodes, I and O
- flow on G_P
- measurement angles ϕ_v



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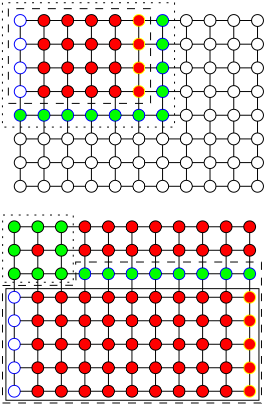
Trappified canvas

- T partial pattern
- σ single-qubit product state on I
- $\mathcal{T}$ an efficiently computable probability distribution for X measurements of qubits it O
- τ a decision algorithms that takes a sample from $\mathcal{T}$ and outputs Pass or Fail



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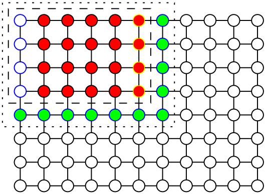
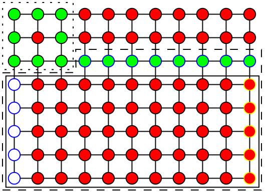
Trappified scheme

- A collection of canvas and an embedding algorithm that maps computations to patterns given a trappified canvas





Conditions for verification

Pauli detection

P ϵ-detects $\mathcal{E} \subset \mathcal{G}_V$ if

$$\forall E \in \mathcal{E}, \sum_{T \in P} \Pr[\tau(t) = 1, T] \geq 1 - \epsilon$$

The probability is over the choice of canvas in the scheme and samples of the trap measurements t

Pauli insensitivity

P is δ-insensitive to $\mathcal{E} \subset \mathcal{G}_V$ if

$$\forall E \in \mathcal{E}, \sum_{T \in P} \Pr[\tau(t) = 0, T] \geq 1 - \delta$$

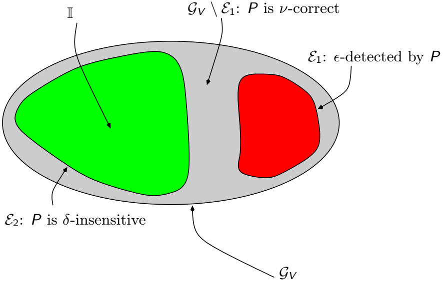
Pauli correctness

P is ν-correct on $\mathcal{E} \subset \mathcal{G}_V$ if,

$$\forall E \in \mathcal{E}, \ \forall C, \ \forall T \in P, \max_{\psi} \|(\tilde{C}_{T,E} - C) \otimes \mathbb{I}_R \, |\psi\rangle\langle\psi| \, \|_{\mathrm{tr}} \leq \nu$$

C is the intended computation, $\tilde{C}_{T,E}$ is the pattern followed by the deviation E

Conditions for verification



Detection implies verifiability

$\mathcal{E}_1$, $\mathcal{E}_2$ two pauli deviations sets with $\mathcal{E}_1 \cap \mathcal{E}_2 = \emptyset$ and $\mathbb{I} \in \mathcal{E}_2$. If P trappified scheme

- ϵ-detects $\mathcal{E}_1$,
- δ-insensitive to $\mathcal{E}_2$,
- ν-correct on $\mathcal{G}_V \setminus \mathcal{E}_1$ P allows for $\delta + \nu$ correct and $\max(\epsilon, \nu)$ secure deletgate quantum computing in AC.

05

Lifting limitations and going toward practical solutions

What is it good for?

Separate concerns

- Trap design is easier and decoupled from the security proof
- Amplification process can be changed

Impact

- Traps based on any measurement of stabilizer generator of the graph work
- Allows to diversify the trappified canvas and adapt them to specific setups
  - Robust verification
  - Multi-party computation
  - Rotation-only clients
  - Fault-tolerant delegation of quantum computation

A simple robust verification protocol

Changing only the amplification procedure

A simple robust verification protocol

Changing only the amplification procedure

- Traps and Computations are in different rounds (test and computation rounds)

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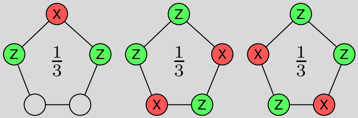
- Test rounds correspond to graph coloring



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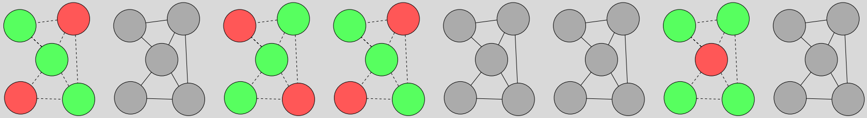

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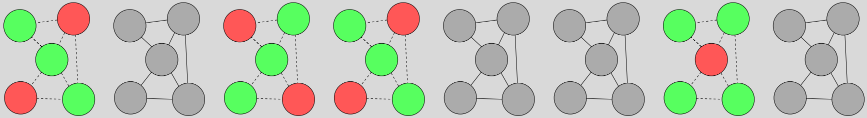

Benefit

- Robust up to 25% failure of test rounds
- Still not scalable (cf. Fault-tolerant version)

Conclusion

1. Practical implementations are possible
2. Protocols scale
3. Open questions
   - Optimized schemes
   - Low overhead verification for sampling
   - Lowering the communication complexity
   - Time to insert verification into HW roadmaps

06

Thanks you! (questions?)

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